## EX20

$f(x) = |3x^2 - 1|.e^{1-x^2}$

1) $\forall x \in Df = IR$ on a $(-x) \in Df = IR$

$f(-x) = |3(-x)^2 - 1|.e^{1-(-x)^2}$

$= |3x^2 - 1|.e^{1-x^2} = f(x)$

D'où $f$ est paire

2) $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} (3x^2 - 1).e^{1-x^2}$

$= \lim\limits_{x \to +\infty} (3x^2 - 1).e.e^{-x^2}$ *posons* $X = -x^2$

$= \lim\limits_{X \to -\infty} (-3X - 1).e.e^{X}$

$= \lim\limits_{X \to -\infty} -3eX.e^{X} - e.e^{X} = 0$

3) $f$ continue sur IR : produit de deux fonctions continues

4)

| $x$ | $-\infty$ | | $-\frac{1}{\sqrt{3}}$ | | $\frac{1}{\sqrt{3}}$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|
| $3x^2-1$ | | $+$ | $0$ | $-$ | $0$ | $+$ | |

* $\lim\limits_{x \to (\frac{1}{\sqrt{3}})^+} \frac{f(x) - f(\frac{1}{\sqrt{3}})}{x - \frac{1}{\sqrt{3}}} = \lim\limits_{x \to (\frac{1}{\sqrt{3}})^+} \frac{(3x^2-1).e^{1-x^2}}{x - \frac{1}{\sqrt{3}}}$

$= \lim\limits_{x \to (\frac{1}{\sqrt{3}})^+} \frac{3(x^2 - (\frac{1}{\sqrt{3}})^2).e^{1-x^2}}{x - \frac{1}{\sqrt{3}}}$

$= \lim\limits_{x \to (\frac{1}{\sqrt{3}})^+} \frac{3(x - \frac{1}{\sqrt{3}}).(x + \frac{1}{\sqrt{3}}).e^{1-x^2}}{x - \frac{1}{\sqrt{3}}}$

$= \lim\limits_{x \to (\frac{1}{\sqrt{3}})^+} 3\left(x + \frac{1}{\sqrt{3}}\right).e^{1-x^2}$

$= 2\sqrt{3}\ e^{\frac{2}{3}} = f'_d\ (\frac{1}{\sqrt{3}})$

* $\lim\limits_{x \to (\frac{1}{\sqrt{3}})^-} \frac{f(x) - f(\frac{1}{\sqrt{3}})}{x - \frac{1}{\sqrt{3}}} =$

$= \lim\limits_{x \to (\frac{1}{\sqrt{3}})^-} -3(x + \frac{1}{\sqrt{3}}).e^{1-x^2}$

$= -2\sqrt{3}\ e^{\frac{2}{3}} = f'_g\ (\frac{1}{\sqrt{3}})$

$f'_g\ (\frac{1}{\sqrt{3}}) \neq f'_d\ (\frac{1}{\sqrt{3}})$ d'où $f$ n'est pas dérivable en $\frac{1}{\sqrt{3}}$

5) $f(x) = \begin{cases} (3x^2-1)e^{1-x^2} \text{ si } x \notin [-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}] \\ -(3x^2-1)e^{1-x^2} \text{ si } x \in [-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}] \end{cases}$

$f'(x) = \begin{cases} 2x(4-3x^2)e^{1-x^2} \text{ si } x \notin [-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}] \\ 2x(3x^2-4)e^{1-x^2} \text{ si } x \in ]-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}[ \end{cases}$

Tableau de variation sur $IR_+$ car $f$ est paire :

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